

1 Platinum nuggets are in the shape of a solid cylinder.

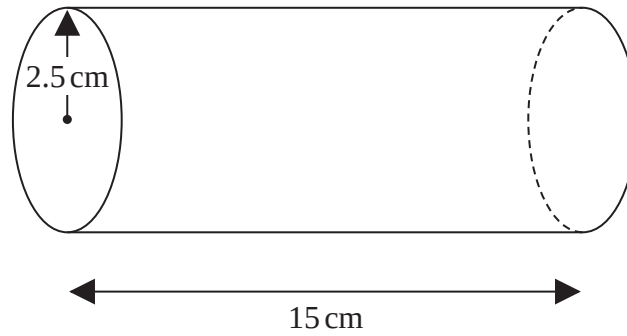


Diagram **NOT**
accurately drawn

The radius of each cylinder is 2.5 cm.

The length of each cylinder is 15 cm.

The density of platinum is 21.5 g/cm^3

The greatest mass that Jacques can carry is 30 kg.

Can Jacques carry 5 platinum nuggets at the same time?

You must show all your working.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume of cylinder} = \pi r^2 h$$

Finding the volume of platinum nugget:

$$\pi \times 2.5^2 \times 15 = 294.52 \text{ cm}^3 \quad (1)$$

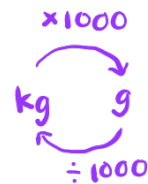
Finding mass of a platinum nugget:

$$21.5 \text{ g/cm}^3 = \frac{\text{mass}}{294.52 \text{ cm}^3} \quad (1)$$

$$\text{mass} = 294.52 \text{ cm}^3 \times 21.5 \text{ g/cm}^3 \quad (1)$$

$$= 6332.27 \text{ g} \div 1000 \leftarrow \text{convert g to kg}$$

$$= 6.33227 \text{ kg}$$



Finding mass of 5 platinum nuggets:

$$5 \times 6.33227 \text{ kg} = 31.661 \text{ kg} > 30 \text{ kg}$$

(1)

∴ No. Jacques cannot carry 5 platinum nuggets

at a time. (1)

(Total for Question 1 is 5 marks)

2 **R** and **S** are two similar solid shapes.

Shape **R** has surface area 108 cm^2 and volume 135 cm^3

Shape **S** has surface area 300 cm^2

Work out the volume of shape **S**.

$$\frac{S}{R} = \sqrt{\frac{300}{108}} = \sqrt[3]{\frac{V_S}{135}} \quad (1)$$

$$V_S = \left(\sqrt{\frac{300}{108}} \right)^3 \times 135 \quad (1)$$

$$= 625 \quad (1)$$

..... 625 cm^3

(Total for Question 2 is 3 marks)

- 3 The diagram shows a solid cylinder with radius 3 m.

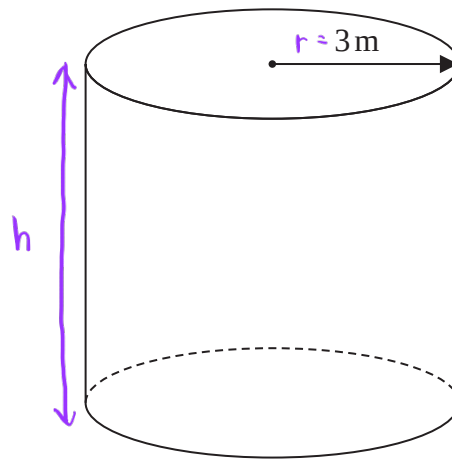


Diagram **NOT**
accurately drawn

The volume of the cylinder is $72\pi\text{ m}^3$

$$\text{Volume of cylinder} = \pi r^2 h$$

Calculate the **total** surface area of the cylinder.
Give your answer correct to 3 significant figures.

$$\text{Volume} = 72\pi = \pi \times 3^2 \times h \quad (1)$$

$$h = \frac{72\pi}{9\pi} = 8\text{ m} \quad (1)$$

$$\begin{aligned} \text{Area of base} &: \pi r^2 = \pi \times 3^2 \\ &= 9\pi \end{aligned}$$

$$\begin{aligned} 2 \text{ bases} &= 2 \times 9\pi \\ &= 18\pi \end{aligned}$$

$$\begin{aligned} \text{Area of lateral face} &= 2 \times \pi \times r \times h \\ &= 2 \times \pi \times 3 \times 8 \\ &= 48\pi \quad (1) \end{aligned}$$

$$\begin{aligned} \text{Total surface area} &= 18\pi + 48\pi \\ &= 66\pi = 207\text{ m}^2 \quad (1) \end{aligned}$$

207

.....m²

(Total for Question 3 is 5 marks)

- 4 The diagram shows a frustum of a cone and a sphere.

The frustum is made by removing a small cone from a large cone.
The cones are similar.

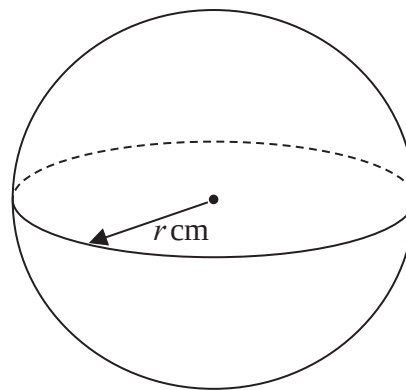
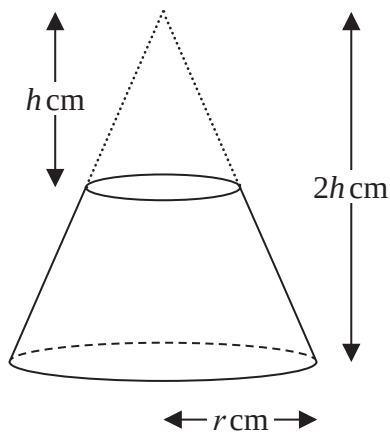


Diagram **NOT**
accurately drawn

The height of the small cone is h cm.
The height of the large cone is $2h$ cm.
The radius of the base of the large cone is r cm.

The radius of the sphere is r cm.

Given that the volume of the frustum is equal to the volume of the sphere,

find an expression for r in terms of h .
Give your expression in its simplest form.

$$\text{Volume of cone : } \frac{1}{3} \pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\begin{aligned} \text{Volume of big cone} &= \frac{1}{3} \pi r^2 (2h) \\ &= \frac{2\pi r^2 h}{3} \quad (1) \end{aligned}$$

$$\begin{aligned} \text{volume of small cone} &= \frac{1}{3} \pi \left(\frac{1}{2} r \right)^2 (h) \\ &= \frac{1}{3} \pi \left(\frac{1}{4} r^2 \right) h \\ &= \frac{1}{12} \pi r^2 h \end{aligned}$$

Volume of frustum = Volume of big cone - volume of small cone

$$\begin{aligned}\text{Volume of frustum} &= \frac{2\pi r^2 h}{3} - \frac{1}{12} \pi r^2 h \\ &= \frac{7}{12} \pi r^2 h \quad (1)\end{aligned}$$

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$(1) \quad \frac{4}{3} \pi r^3 = \frac{7}{12} \pi r^2 h \quad (1)$$

$$\frac{4}{3} r = \frac{7}{12} h$$

$$4 \times 12 r = 7 \times 3 h$$

$$48 r = 21 h$$

$$r = \frac{21 h}{48}$$

$$r = \frac{7}{16} h$$

$$r = \frac{7}{16} h \quad (1)$$

(Total for Question 4 is 5 marks)

5

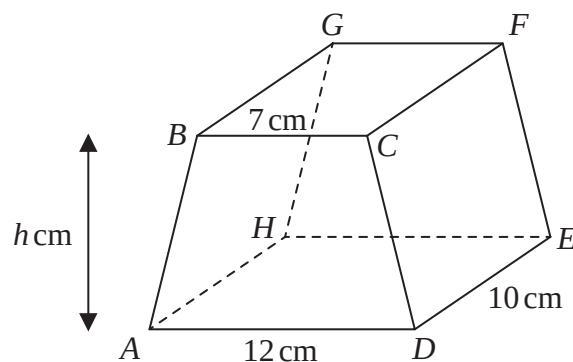


Diagram **NOT**
accurately drawn

The diagram shows a prism $ABCDEFGH$ in which $ABCD$ is a trapezium with BC parallel to AD and $CDEF$ is a rectangle.

$$BC = 7 \text{ cm} \quad AD = 12 \text{ cm} \quad DE = 10 \text{ cm}$$

The height of trapezium $ABCD$ is $h \text{ cm}$

The volume of the prism is 608 cm^3

Work out the value of h .

$$\text{Volume} = \text{area of trapezium} \times \text{width}$$

$$= \frac{1}{2} \times (BC + AD) \times h \times DE$$

$$\text{Volume} = \frac{1}{2} \times (7 + 12) \times h \times 10 = 608 \quad (1)$$

$$: 95h = 608$$

$$h = \frac{608}{95} \quad (1)$$

$$= 6.4 \quad (1)$$

$$h = \underline{\quad 6.4 \quad}$$

(Total for Question 5 is 3 marks)

6 The diagram shows two similar vases, **A** and **B**.

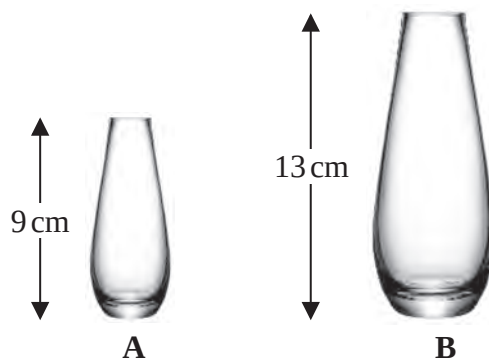


Diagram **NOT**
accurately drawn

The height of vase **A** is 9 cm and the height of vase **B** is 13 cm.

Given that

$$\text{surface area of vase A} + \text{surface area of vase B} = 1800 \text{ cm}^2$$

calculate the surface area of vase **A**.

Comparing scale factor of A and B :

$$\begin{array}{lcl} & A & : B \\ \text{Height} & 9 & : 13 \\ & 9^2 & : 13^2 \\ \text{Area} & 81 & : 169 \end{array}$$

$$\therefore \frac{A}{B} = \frac{81}{169} \quad (1)$$

$$A + B = 1800$$

$$A + \frac{169}{81} A = 1800 \quad (1)$$

$$\frac{250}{81} A = 1800 \quad (1)$$

$$A = 583.2 \quad (1)$$

$$\dots\dots\dots 583.2 \text{ cm}^2$$

(Total for Question 6 is 4 marks)

7 The diagram shows a solid cube.

The cube is placed on a table so that the whole of one face of the cube is in contact with the table.

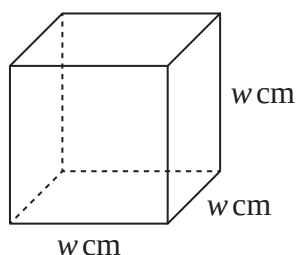


Diagram **NOT**
accurately drawn

The cube exerts a force of 56 newtons on the table.

The pressure on the table due to the cube is 0.14 newtons/cm²

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the volume of the cube.

$$0.14 \text{ N/cm}^2 = \frac{56 \text{ N}}{w^2} \quad (1)$$

$$w^2 = \frac{56}{0.14}$$

$$w^2 = 400$$

$$w = \sqrt{400}$$

$$= 20 \text{ cm} \quad (1)$$

$$\text{Volume of cube} = 20 \text{ cm} \times 20 \text{ cm} \times 20 \text{ cm} \quad (1)$$

$$= 8000 \text{ cm}^3 \quad (1)$$

8000

..... cm³

(Total for Question 7 is 4 marks)

8 Here is a sector, AOB , of a circle with centre O and angle $AOB = x^\circ$

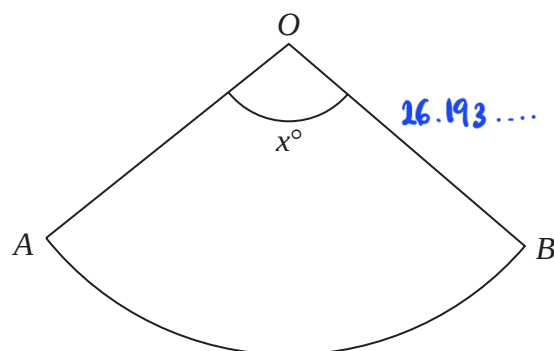


Diagram **NOT**
accurately drawn

The sector can form the curved surface of a cone by joining OA to OB .

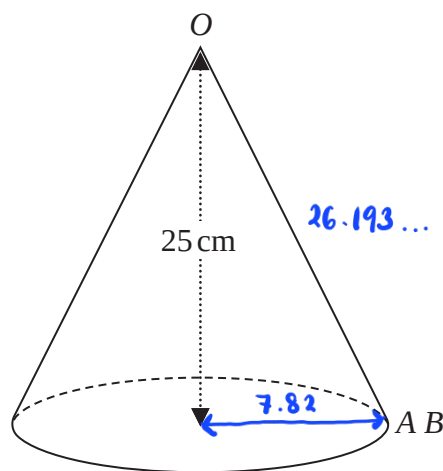


Diagram **NOT**
accurately drawn

The height of the cone is 25 cm.

The volume of the cone is 1600 cm^3

Work out the value of x .

Give your answer correct to the nearest whole number.

Volume of cone :

$$\frac{1}{3} \times \pi \times r^2 \times h$$

Finding radius of the cone :

$$\frac{1}{3} \times \pi \times r^2 \times 25 = 1600 \quad (1)$$

$$\pi r^2 = \frac{1600}{25} \times 3$$

$$r^2 = \frac{192}{\pi}$$

$$r = \sqrt{61.116}$$

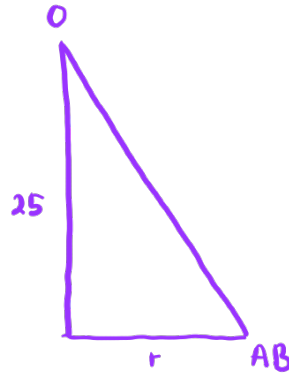
$$= 7.8176 \dots \text{ cm} \quad (1)$$

By using Pythagoras' theorem :

$$OA^2 = 25^2 + 7.8176^2 \dots$$

$$OA = \sqrt{686.1154 \dots}$$

$$= 26.193 \dots \quad (1)$$



circumference of the circle :

$$2\pi r = 2 \times \pi \times 7.8176 \dots$$

$$= 49.1194 \dots \quad (1)$$

length of arc of the circle :

$$2 \times \pi \times 26.193 \dots \times \frac{x}{360^\circ} = 49.1194 \dots \quad (1)$$

$$x = 107^\circ \quad (1)$$

$$x = \dots 107^\circ \dots$$

(Total for Question 8 is 6 marks)

9 Here is a triangular prism.

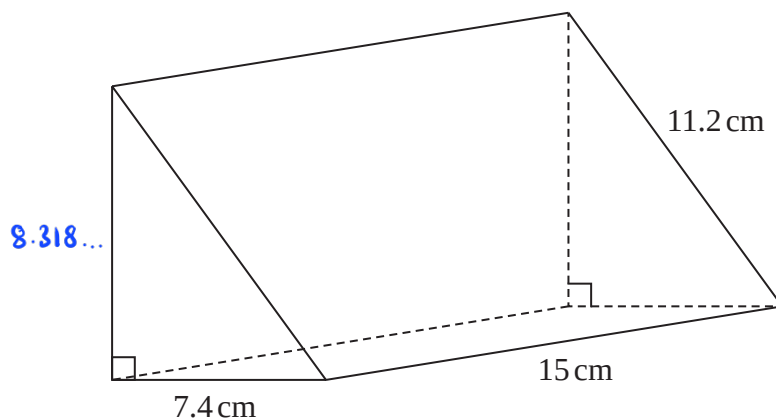


Diagram **NOT**
accurately drawn

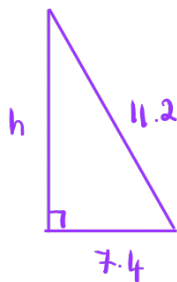
Work out the volume of the prism.

Give your answer correct to 3 significant figures.

cross section of the prism :

$$h = \sqrt{11.2^2 - 7.4^2} \quad (1)$$

$$= 8.407... \quad (1)$$



Area of cross section :

$$\frac{1}{2} \times 7.4 \times 8.407...$$

$$= 31.106... \quad (1)$$

Volume of prism = Area of cross section $\times$ length

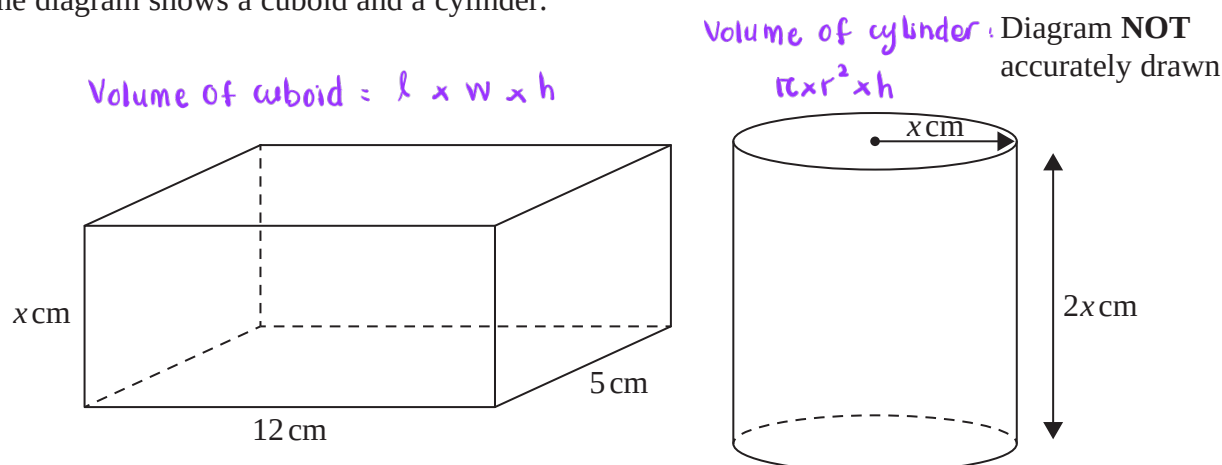
$$= 31.106... \times 15 \quad (1)$$

$$= 467 \quad (1)$$

..... 467 cm³

(Total for Question 9 is 5 marks)

- 10 The diagram shows a cuboid and a cylinder.



The dimensions of the cuboid are x cm by 12 cm by 5 cm.
The volume of the cuboid is 270 cm^3

The radius of the cylinder is x cm.
The height of the cylinder is $2x$ cm.

- (a) Work out the volume of the cylinder.
Give your answer correct to the nearest whole number.

$$\text{Volume of cuboid} = 12 \times 5 \times x = 270$$

$$60x = 270$$

$$x = \frac{270}{60}$$

$$= 4.5 \text{ cm } \textcircled{1}$$

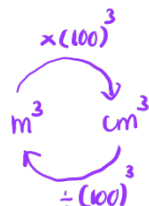
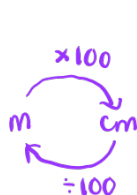
$$\text{Volume of cylinder} = \pi \times x^2 \times 2x$$

$$= \pi \times (4.5)^2 \times 2(4.5) \textcircled{1}$$

$$= 573 \text{ cm}^3 \textcircled{1}$$

$$\begin{array}{r} 573 \\ \hline \end{array} \text{ cm}^3 \quad (3)$$

- (b) Change 1 m^3 to cm^3

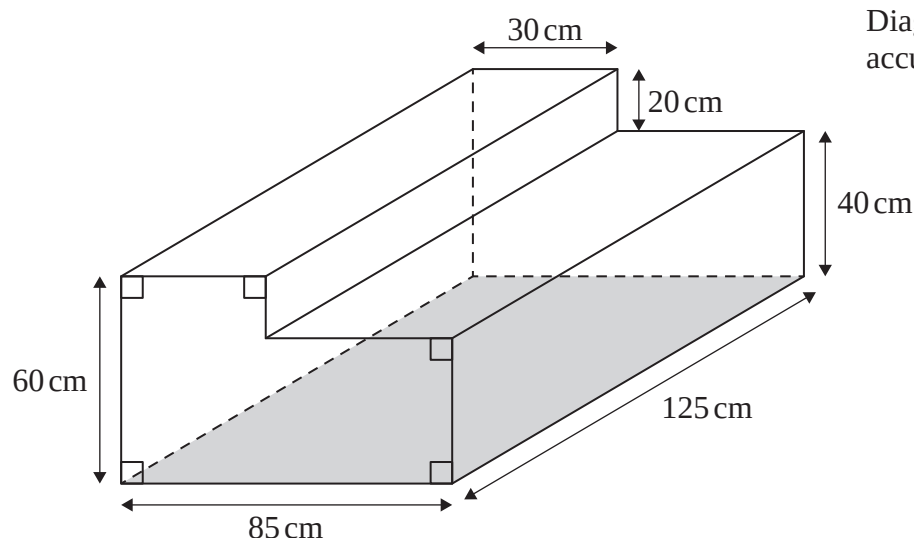


$$1 \text{ m}^3 \times \frac{(100)^3 \text{ cm}^3}{(1)^3 \text{ m}^3} = 1\,000\,000 \textcircled{1}$$

$$\begin{array}{r} 1\,000\,000 \\ \hline \end{array} \text{ cm}^3 \quad (1)$$

(Total for Question 10 is 4 marks)

- 11 The diagram shows a container for water in the shape of a prism.



The rectangular base of the prism, shown shaded in the diagram, is horizontal.
The container is completely full of water.

Tuah is going to use a pump to empty the water from the container so that the volume of water in the container decreases at a constant rate.

The pump starts to empty water from the container at 10 30 and at 12 00 the water level in the container has dropped by 20 cm.

Find the time at which all the water has been pumped out of the container.

$$85 \times 125 \times 40 = 425\,000 \text{ cm}^3 \quad (\text{water left in container})$$

$$\textcircled{1} \quad 30 \times 20 \times 125 = 75\,000 \text{ cm}^3 \quad (\text{water that has been pumped out})$$

$$\frac{75\,000 \text{ cm}^3}{425\,000 \text{ cm}^3} = \frac{1.5 \text{ hour}}{x}$$

$$x = \frac{425\,000 \times 1.5}{75\,000} \quad \textcircled{2}$$

$$= 8.5 \text{ hours}$$

$$1200 + 8.5 \text{ hours} = 2030 \quad \textcircled{1}$$

2030

(Total for Question 11 is 4 marks)

12 A solid, **S**, is made from a hemisphere and a cylinder.

The centre of the circular face of the hemisphere and the centre of the top face of the cylinder are at the same point.

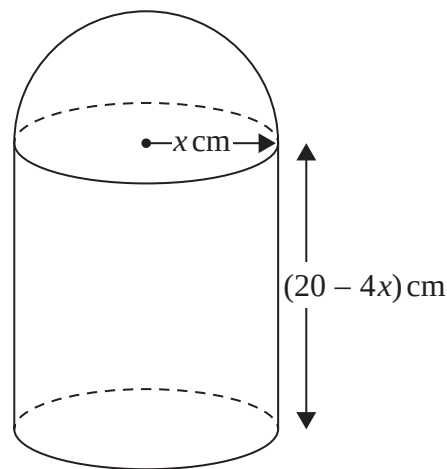


Diagram **NOT**
accurately drawn

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $(20 - 4x)$ cm.

The volume of **S** is $V \text{ cm}^3$ where $V = \frac{1}{3} \pi y$

Find the maximum value of y .
Show clear algebraic working.

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3 \quad \text{Volume of hemisphere} = \frac{2}{3} \pi r^3$$

$$\begin{aligned} \text{Volume of S} &= \frac{2}{3} \pi r^3 + \pi r^2 h \\ &= \frac{2}{3} \pi x^3 + \pi x^2 (20 - 4x) \\ &= \frac{2}{3} \pi x^3 + 20 \pi x^2 - 4 \pi x^3 \quad \textcircled{1} \end{aligned}$$

$$\frac{1}{3} \pi y = \frac{2}{3} \pi x^3 + 20 \pi x^2 - 4 \pi x^3 \quad \textcircled{1}$$

$$\frac{1}{3} \pi y = \left(\frac{2}{3} \pi - 4 \pi \right) x^3 + 20 \pi x^2 \quad (\text{cancel out } \pi)$$

$$\frac{1}{3} y = \left(\frac{2}{3} - 4 \right) x^3 + 20 x^2$$

$$= -\frac{10}{3} x^3 + 20 x^2$$

$$\frac{y}{3} = -\frac{10}{3}x^3 + 20x^2$$

$$y = -10x^3 + 60x^2 \quad (1)$$

$$\frac{dy}{dx} = -30x^2 + 120x$$

$$0 = -30x^2 + 120x \quad (1)$$

$$30x^2 = 120x$$

$$x = \frac{120x}{30x}$$

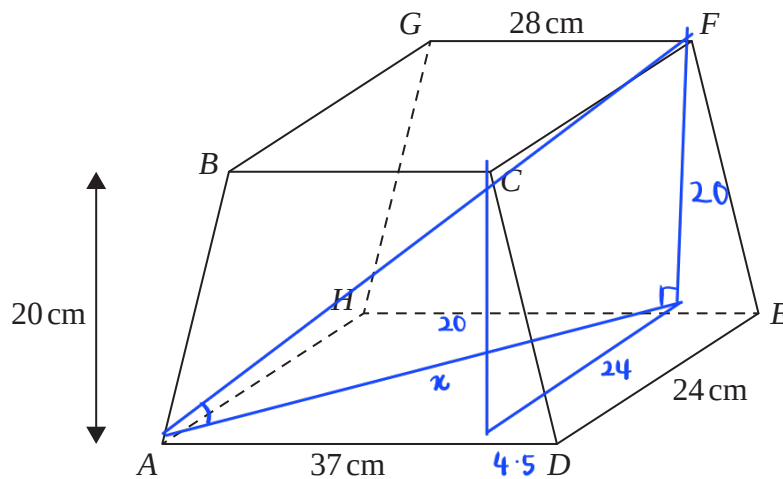
$$x = 4$$

$$y = -10(4)^3 + 60(4)^2$$
$$= 320 \quad (1)$$

320

(Total for Question 12 is 5 marks)

Diagram **NOT**
accurately drawn



(a) Work out the total surface area of the prism.

$$\begin{aligned} \text{Total surface area} &= 2 \times \frac{1}{2} \times (37+28) \times 20 + 2 \times 24 \times 20 + 28 \times 24 + \\ &\quad 24 \times 37 \\ &= 1300 + 984 + 672 + 888 \\ &= 3844 \text{ cm}^2 \end{aligned}$$

~~(4)~~

(Total for Question 13 is 4 marks)

14 The diagram shows a frustum of a cone, and a sphere.

The frustum, shown shaded in the diagram, is made by removing the small cone from the large cone.

The small cone and the large cone are similar.

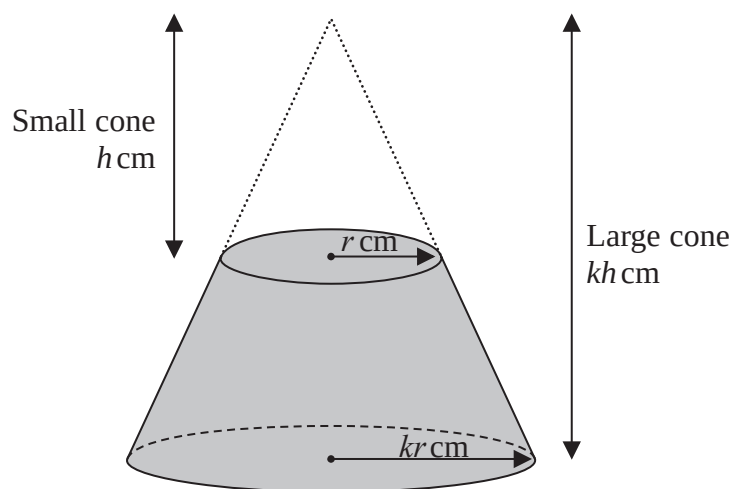
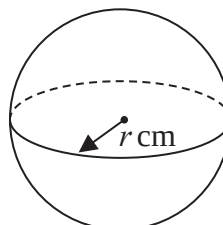


Diagram **NOT** accurately drawn



The height of the small cone is h cm and the radius of the base of the small cone is r cm.
The height of the large cone is kh cm and the radius of the base of the large cone is kr cm.
The radius of the sphere is r cm.

The sphere is divided into two hemispheres, each of radius r cm.

Solid **A** is formed by joining one of the hemispheres to the frustum.

The plane face of the hemisphere coincides with the upper plane face of the frustum, as shown in the diagram below.

Solid **B** is formed by joining the other hemisphere to the small cone that was removed from the large cone.

The plane face of the hemisphere coincides with the plane face of the base of the small cone, as shown in the diagram below.

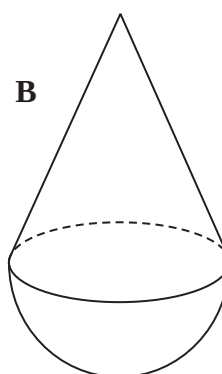
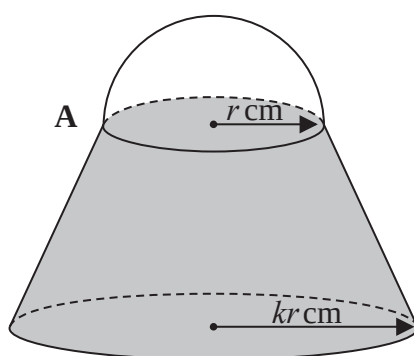


Diagram **NOT** accurately drawn

The volume of solid **A** is 6 times the volume of solid **B**.

Given that $k > \sqrt[3]{7}$

find an expression for h in terms of k and r

Volume of each hemisphere :

$$\begin{aligned}\frac{1}{2} \times \text{volume of sphere} &= \frac{1}{2} \times \frac{4}{3} \times \pi \times r^3 \\ &= \frac{2}{3} \pi r^3 \quad (1)\end{aligned}$$

Volume of small cone :

$$= \frac{1}{3} \pi r^2 h$$

Volume of frustrum :

Volume of large cone - Volume of small cone :

$$\begin{aligned}\frac{1}{3} \times \pi \times (kr)^2 \times kh - \frac{1}{3} \times \pi \times r^2 \times h \\ = \frac{1}{3} \pi r^2 h (k^3 - 1) \quad (1)\end{aligned}$$

Volume of Solid A :

$$= \frac{1}{3} \pi r^2 h (k^3 - 1) + \frac{2}{3} \pi r^3$$

Volume of Solid B :

$$= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \quad (1)$$

$$\therefore \frac{1}{3} \pi r^2 h (k^3 - 1) + \frac{2}{3} \pi r^3 = 6 \left(\frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \right) \quad (1)$$

$$\frac{1}{3} \pi r^2 h (k^3 - 1) + \frac{2}{3} \pi r^3 = 2 \pi r^2 h + 4 \pi r^3$$

$$\frac{1}{3} h (k^3 - 1) + \frac{2}{3} r = 2h + 4r$$

$$h(k^3 - 1) + 2r = 6h + 12r$$

$$h(k^3 - 1) - 6h = 10r \quad (1)$$

$$hk^3 - 7h = 10r$$

$$h = \frac{10r}{k^3 - 7} \quad (1)$$

$$h = \frac{10r}{k^3 - 7}$$

(Total for Question 14 is 6 marks)

15 A solid is made from a cone and a hemisphere.

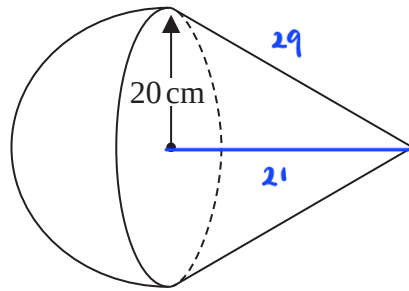


Diagram **NOT**
accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm.

The curved surface area of the cone is $580\pi\text{ cm}^2$

The volume of the solid is $k\pi\text{ cm}^3$

Work out the exact value of k

$$580\pi = \pi \times 20 \times l \quad (1)$$

$$l = \frac{580\pi}{20\pi}$$

$$= 29 \quad (1)$$

$$= \sqrt{29^2 - 20^2} = 21 \quad (1)$$

$$\left(\frac{1}{2} \times \frac{4}{3} \times \pi \times 20^3 \right) + \left(\frac{1}{3} \times \pi \times 20^2 \times 21 \right)$$

$$= \frac{16000}{3}\pi + \frac{8400}{3}\pi \quad (1)$$

$$= \frac{24400}{3}\pi \quad (1)$$

$$k = \frac{24400}{3}$$

(Total for Question 15 is 5 marks)

- 16 The diagram shows a solid triangular prism.

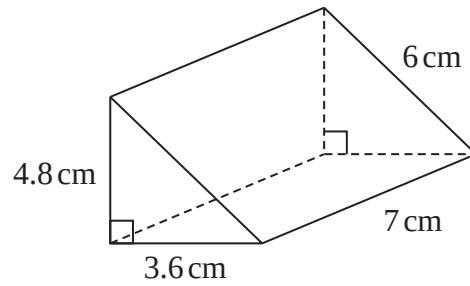


Diagram **NOT**
accurately drawn

Work out the **total** surface area of the triangular prism.
Give your answer correct to 3 significant figures.

$$\begin{aligned}
 & \left(2 \times \frac{1}{2} \times 4.8 \times 3.6 \right) + (7 \times 6) + (7 \times 3.6) + (4.8 \times 7) \\
 &= 17.28 + 42 + 25.2 + 33.6 \\
 &= 118.08 \\
 &\approx 118
 \end{aligned}$$

118 cm²

(Total for Question 16 is 3 marks)

17 The diagram shows two solids, **A** and **B**, made from two different metals.

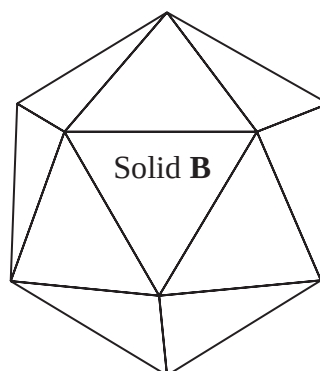
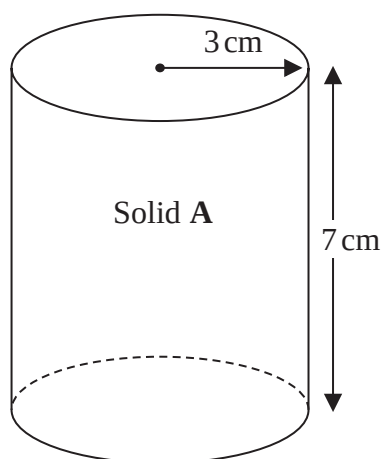


Diagram **NOT**
accurately drawn

Solid **A** is in the shape of a cylinder with radius 3 cm and height 7 cm
Solid **A** has a mass of 2000 g

Solid **B** has a mass of 3375 g
Solid **B** has a volume of 450 cm^3

All of the metal from Solid **A** and Solid **B** is melted down to make a uniform Solid **C**

Given that there is no change to mass or volume during this process

work out the density of Solid **C**

Give your answer correct to one decimal place.

$$\text{volume A} : \pi \times 3^2 \times 7 = 197.9 \dots \text{ (1)}$$

$$\text{density C} : \frac{2000 + 3375}{197.9 \dots + 450} \text{ (1)}$$

$$= 8.3 \text{ (1)}$$

..... **8.3** g/cm^3

(Total for Question 17 is 3 marks)

18 A statue and a model of the statue are mathematically similar.

The statue has a total surface area of 3600 cm^2

The model has a total surface area of 625 cm^2

The volume of the model is 750 cm^3

Work out the volume of the statue.

length scale factor :

$$\sqrt{\frac{3600}{625}} = \frac{60}{25} = \frac{12}{5} \quad (1)$$

$$\text{volume of statue} = \left(\frac{12}{5}\right)^3 \times 750 \quad (1)$$

$$= \frac{1728}{125} \times 750$$

$$= 10\,368 \quad (1)$$

10 368 cm^3

(Total for Question 18 is 3 marks)

19 The diagram shows a solid cone and a solid sphere.

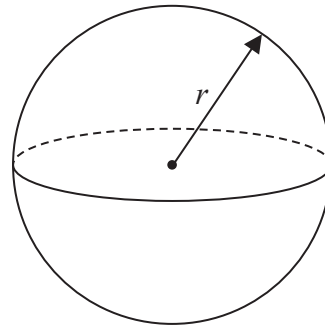
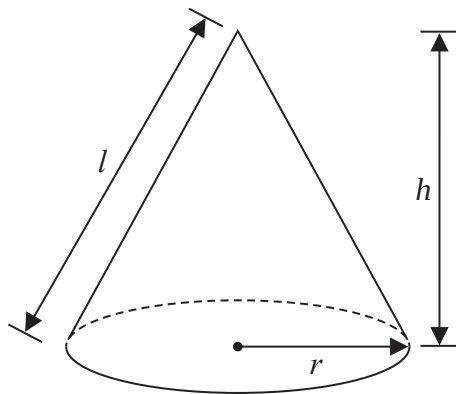


Diagram **NOT** accurately drawn

The cone has base radius r , slant height l and perpendicular height h
 The sphere has radius r

The base radius of the cone is equal to the radius of the sphere.

Given that

$$k \times \text{volume of the cone} = \text{volume of the sphere}$$

show that the **total** surface area of the cone can be written in the form

$$\pi r^2 \left(\frac{k + \sqrt{k^2 + a}}{k} \right)$$

where a is a constant to be found.

$$k \times \frac{1}{3} \times \cancel{\pi} \times r^2 \times h = \frac{4}{3} \times \cancel{\pi} \times r^3 \quad (1)$$

$$kh = 4r$$

$$h = \frac{4r}{k} \quad (1)$$

$$l^2 = h^2 + r^2$$

$$= \left(\frac{4r}{k} \right)^2 + r^2 \quad (1)$$

$$l = \sqrt{r^2 + \left(\frac{4r}{k} \right)^2}$$

$$l = \sqrt{r^2 \left(1 + \frac{16}{k^2}\right)}$$

$$l = r \sqrt{\frac{k^2 + 16}{k^2}}$$

$$l = r \frac{\sqrt{k^2 + 16}}{k} \quad (1)$$

$$\text{Total surface area} : \pi r^2 + \pi r l$$

$$= \pi r^2 \left(1 + \frac{\sqrt{k^2 + 16}}{k}\right) \quad (1)$$

$$= \pi r^2 \left(\frac{k + \sqrt{k^2 + 16}}{k}\right) \quad (1)$$

(Total for Question 19 is 6 marks)

20 Given that the surface area of a sphere is $49\pi \text{ cm}^2$

find the volume of the sphere.

Give your answer correct to the nearest integer.

$$4\pi r^2 = 49\pi$$

$$r^2 = \frac{49\pi}{4\pi} \quad (1)$$

$$r = \sqrt{\frac{49\pi}{4\pi}}$$

$$= \frac{7}{2} = 3.5$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 3.5^3 \quad (1)$$

$$= 180 \quad (1)$$

..... 180 cm^3

(Total for Question 20 is 3 marks)

- 21 The diagram shows a rectangular sheet of metal $ABCD$

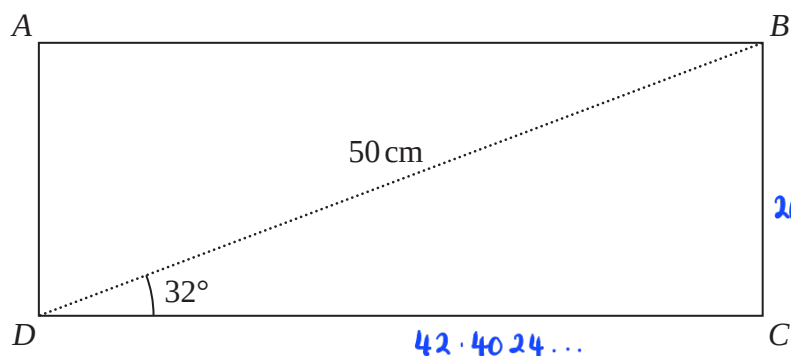


Diagram **NOT**
accurately drawn

$BD = 50$ cm and angle $BDC = 32^\circ$

Nasser joins side AD to side BC to form a cylinder.

BC is the height of the cylinder.

DC is the circumference of the cross section of the cylinder.

Work out the volume, in cm^3 , of the cylinder.

Give your answer correct to 3 significant figures.

$$\sin 32^\circ = \frac{BC}{50} \quad (1)$$

$$BC = 50 \sin 32^\circ = 26.4959... \quad (1)$$

$$\cos 32^\circ = \frac{CD}{50} \quad (1)$$

$$CD = 50 \cos 32^\circ = 42.4024...$$

$$42.4024... = 2\pi r$$

$$r = \frac{42.4024...}{2\pi} = 6.74855... \quad (1)$$

$$\text{Volume} = \pi \times 6.74855...^2 \times 26.4959... \quad (1)$$

$$= 3796 \quad (1)$$

3 790

..... cm³

(Total for Question 21 is 6 marks)

- 22 A frustum is made by removing a small square-based pyramid from a similar large squared-based pyramid as shown in the diagram.

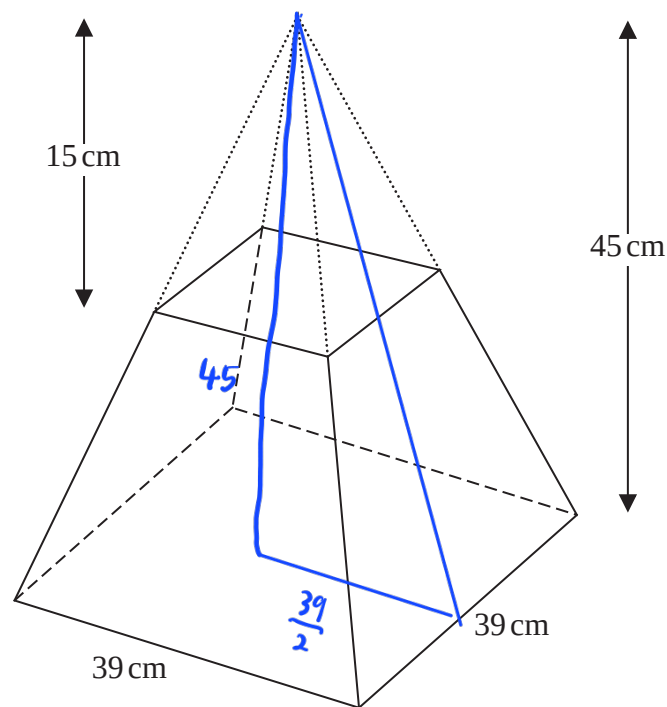


Diagram **NOT** accurately drawn

The height of the small pyramid is 15 cm.

The height of the large pyramid is 45 cm.

The square base of the large pyramid has side length 39 cm.

Work out the **total** surface area of the frustum.

Give your answer correct to the nearest whole number.

$$\text{Area of small square} = \left(\frac{15}{45} \times 39 \right)^2 = 13^2 = 169$$

$$\text{Area of large square} = 39^2 = 1521 \quad (1)$$

$$\begin{aligned} \text{slant height of large pyramid} &= \sqrt{45^2 + \left(\frac{39}{2} \right)^2} \\ &= 49.043 \dots \quad (1) \end{aligned}$$

$$\text{slant height of small pyramid} = 49.043 \times \frac{15}{45} = 16.347 \dots$$

$$\text{Area of large slanted triangles} : 4 \times \frac{1}{2} \times 49.043 \times 39 = 3825.381 \quad (1)$$

$$\text{Area of small slanted triangles} : 4 \times \frac{1}{2} \times 16.347 \times 13 = 425.042$$

$$\text{Area of slanted surfaces (frustum)} : 3825.381 - 425.042 = 3400.339 \dots$$

$$\begin{aligned}\text{Total surface area} &= 169 + 1521 + 3400 \cdot 339 \text{ (1)} \\ &= 5090 \cdot 339 \dots \\ &= 5090 \text{ (1)}\end{aligned}$$

..... 5090 cm^2

(Total for Question 22 is 5 marks)

23 The diagram shows a cuboid with a square cross section.

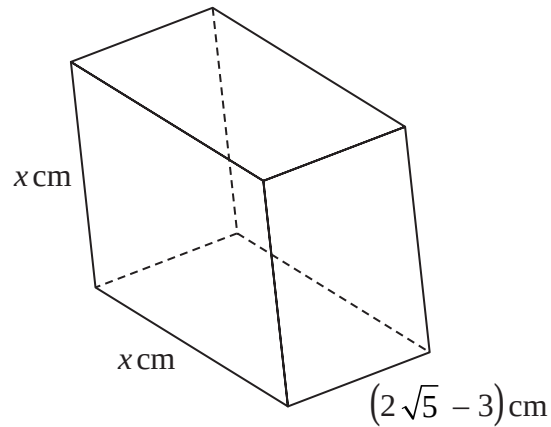


Diagram **NOT**
accurately drawn

The volume of the cuboid is $(13 + 6\sqrt{5})\text{cm}^3$

Without using a calculator, find the value of x

Give your answer in the form $a + \sqrt{b}$ where a and b are integers.

Show your working clearly.

$$x \times x \times (2\sqrt{5} - 3) = 13 + 6\sqrt{5}$$

$$x^2 = \frac{13 + 6\sqrt{5}}{2\sqrt{5} - 3} \times \frac{2\sqrt{5} + 3}{2\sqrt{5} + 3}$$

$$= \frac{26\sqrt{5} + 39 + 12(5) + 18\sqrt{5}}{4(5) - 9}$$

$$= \frac{39 + 60 + 26\sqrt{5} + 18\sqrt{5}}{11}$$

$$= \frac{99 + 44\sqrt{5}}{11}$$

$$x^2 = 9 + 4\sqrt{5}$$

$$x = a + \sqrt{b}$$

$$x^2 = a^2 + 2a\sqrt{b} + b$$

$$a^2 + b = 9$$

$$2a\sqrt{b} = 4\sqrt{5}$$

$$2a = 4$$

$$a = 2, b = 5$$

$$x = 2 + \sqrt{5} \quad (1)$$

(Total for Question 23 is 4 marks)